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COMPUTATIONAL SOLUTION OF RANDOM INTEGRAL EQUATIONS.(U)
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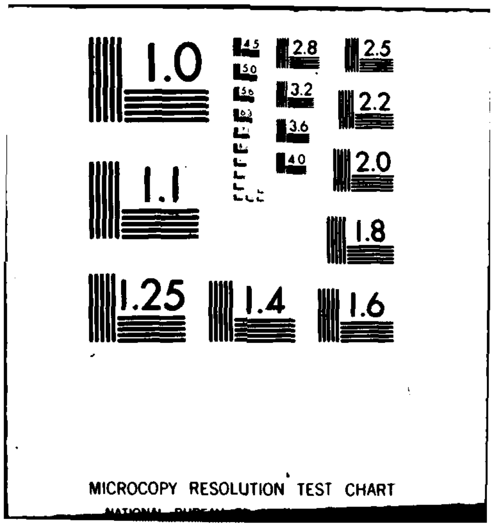
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MICROCOPY RESOLUTION TEST CHART

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This research project was concerned with the initial stages of the systematic development of approximate methods (analytical and computational) for the solution of random integral equations.

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Statement of Problem Studied

This research project was concerned with the initial stages of the systematic development of computational methods for the solution of random integral equations. Our main emphasis was on fixed point and other iterative methods, and degenerate kernel methods.

Because many numerical routines for the solution of random integral equations lead to systems of random linear equations, random matrices and their random characteristic polynomials were studied. Approximate solution of random integral equations also led to the study of approximate probability measures (solution measures) and their convergence properties.

Summary of Results

At the time this project was initiated very few studies had been carried out on the numerical solution of random integral equations. The use of fixed point methods to establish the existence, uniqueness, and measurability of solutions of Itô random integral equations, as well as probabilistic analogues of Fredholm, Volterra, and Hammerstein integral equations, led to approximate solutions of these equations; however it is only recently that computational (or numerical) methods have been employed to solve random integral equations of the above types.

Our research efforts were devoted exclusively to random Fredholm integral equations of the second kind. Our first step was to reexamine the literature devoted to the numerical solution of deterministic Fredholm integral equations, with particular reference to those numerical routines which gave the 'best' (in some appropriate sense) solution. Because there were very few results known about the modification of numerical routines for the solution of random Fredholm integral equations, our first studies were of any experimental or exploratory nature. Since most iterative-projection methods for the solution of integral equations lead to linear systems of algebraic equations, our first theoretical studies were concerned with the behavior of the roots of random algebraic polynomials which arise as the characteristic polynomials of the random matrices associated with linear systems of random algebraic equations.

Let $F_n(z,\Omega)$ be a random algebraic polynomial of degree n; that is an algebraic polynomial of degree n with random coefficients. Publication No. 1 (see List of Publications) considered the behavior of the roots of random algebraic polynomials. A code was developed which (1) generates a sample of random algebraic polynomials, (2) calculates the roots of each sample polynomial, and then (3) calculates the averages of the roots. Finally, the roots of the deterministic algebraic polynomial whose coefficients are the averages of the sample coefficients were calculated. The relationship between the averages of the roots of the sample polynomials and the roots of the average polynomial was examined. In particular, we proved a strong law of large numbers for the sample roots; and obtained an estimate of the difference between the averages of the roots of the sample polynomials and the roots of the average polynomials.

As a well-known, associated with any algebraic polynomial is a matrix known as its companion matrix; and the eigenvalues of the associated companion matrix are the roots of the algebraic polynomial. This i obvious theoretical result; however it was not obvious what differen if any, would arise if one investigated the eigenvalues of the random companion matrices associated with random algebraic polynomials. In Publication No. 2 we examined this question, and found very little difference between the roots of the random polynomials and the random matrices.

As is well-known in applied mathematics, Newton's method is utilized to obtain approximate solutions of a number of classes of linear and nonlinear operator equations. In Publication No. 3 we gave two formulations of Newton's iterative process for the solution of random operator equations.

These algorithms are now be implemented on a computer; and the resulting programs will be used to obtain numerical solutions of random integral equations.

A well-defined random function has associated with it a probability measure on some function space; hence when we 'solve' random equations there is an associated probability measure, called the solution measure, induced the random function that is the solution of the equation. Hence, when we generate approximate random solutions of a random equation we have an associated sequence of approximate solution measures. In Publications 4 and 7 we give a sufficient condition for the weak compactness of a sequence of approximate solution measures. Results of this type are required in order to study the weak convergence of the approximate solution measures.

Publication 5 and 6 are the first of a series of papers to deal explicitly with the solution of probabilistic analogues of classical integral equations. In these two papers we consider random Fredholm equations of the second kind. In Publication No. 5 we gave algorithms for the solution of equations of this type. These algorithms are based on the program IESIMP of K. E. Atkinson. Fredholm equation with random right-hand side or random kernels, or both, were considered. Numerical results are presented showing the behavior of the averaged approximate solutions, as well as the variance of these solutions. In Publication No. 6 we considered the special, but important, case of Fredholm equations with random degenerate kernels. In particular, the distribution of the random eigenvalues of the associated random linear algebraic system was considered.

List of Publications

1. M. J. Christensen and A. T. Bharucha-Reid, Stability of the roots or random algebraic polynomials, Commun. Statist.-Simula. Computa. B9 (1980), 179-197.

2. M. J. Christensen and A. T. Bharucha-Reid, Companion matrices associated with random algebraic polynomials, in P. R. Krishnaiah (Editor), "Multivariate Analysis-V," pp. 265-272, North-Holland, Amsterdam, New York, Oxford, 1980.

3. A. T. Bharucha-Reid and R. Kannan, Newton's method for random operator equations, Nonlinear Analysis 4 (1980), 231-240.

4. A. T. Bharucha-Reid, Weak compactness of solution measures associated with random equations, in A. Beck (Editor), "Probability in Banach Spaces III," pp. 25-34, Springer-Verlag, Berlin, Heidelberg, New York, 1981.

5. M. J. Christensen and A. T. Bharucha-Reid, Numerical solution of random integral equations. I. Fredholm equations of the second kind, J. Integral Equations 3 (1981), 217-229.

6. M. J. Christensen and A. T. Bharucha-Reid, Numerical solution of random integral equations. II. Fredholm equations with degenerate kernels, J. Integral Equations 3 (1981), 333-344.

7. A. T. Bharucha-Reid and R. Kannan, Weak compactness of probability measures associated with random equations: I, to appear in Ann. Inst. H. Poincaré, Sect. B.

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